Objectives:

- Review inverse functions and define inverse trigonometric functions.
- Use implicit differentiation to find derivatives of inverse trigonometric functions.

Review of Inverse Functions:

To find the inverse of a function, reverse the roles of input and output.
If $f(a) = b$ then ________________ .

If $f(a) = f(c) = b$ for some $a \neq c$, we have a problem—what should $f^{-1}(b)$ be?

So, a function is only invertible if each input has a unique output. If a function has this nice quality, we say it is ________________ .

Graphically, a function is invertible if it passes the ______________________ test.

If f is invertible and $f(a) = b$, then $f^{-1}\big(f(a)\big) =$ ____________ and $f\big(f^{-1}(b)\big) =$ ____________ .

Inverse Trigonometric Functions:

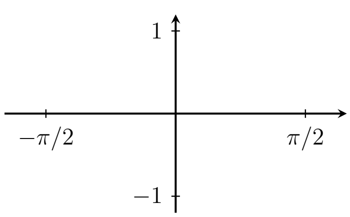
Let's say we want to find an inverse function for $f(x) = \sin(x)$,

which we will call ______________________ .

First, we need to restrict the domain of $\sin(x)$ so that we have an invertible function:

The domain we choose is ________________ .

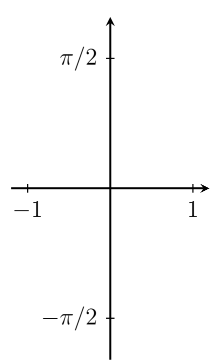
Note: Why choose this domain?



$\sin(x)$, restricted

Domain:

Range:



$\arcsin(x)$

Domain:

Range:

Another important way of understanding this function is that $y = \arcsin(x)$ means:

AND

Now we have a new function! How can we use it?

What should the angles be in a right triangle with a hypotenuse of 3 cm and one side that is $\sqrt{3}$ cm?

Suppose the position of a particle is given by $s(t) = \sin(t)$. When is the particle at position $\frac{1}{2}$?

The derivative of $\arcsin(x)$**:**

To find the derivative of $\arcsin(x)$, we're going to use our sneaky technique of implicit differentiation:

First: Rewrite the equation for $\arcsin(x)$ in terms of $\sin(x)$:

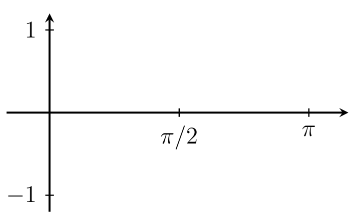
Second: Differentiate both sides:

Third: Solve for $\frac{dy}{dx}$

Fourth: Substitute to find a way to express $\frac{dy}{dx}$ in terms of x.

In today's activity you'll see another way to use implicit differentiation to find the derivative of a trigonometric function.

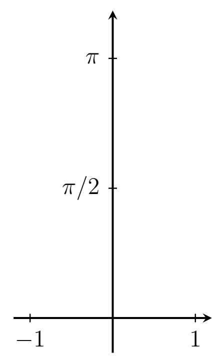
The last inverse trigonometric function we'll see is $\arccos(x)$:



$\cos(x)$, restricted

Domain:

Range:



$\arccos(x)$

Domain:

Range:

To find the derivative, we'll use implicit differentiation again.

Derivatives of Inverse Trigonometric Functions:

$$\frac{d}{dx}\big(\,\arcsin(x)\big) =$$

$$\frac{d}{dx}\big(\,\arccos(x)\big) =$$

$$\frac{d}{dx}\big(\,\arctan(x)\big) =$$

Examples:

(a) Find the derivative of $f(x) = \arctan\left(x^3\right)$.

(b) $y = x^2 e^{\arcsin(x)}$. Find y'.